

Asymptotic Stability of Delay-Difference Control System of Cellular Neural Networks via Matrix Inequalities and Application

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Abstract:

In this paper, we derive a sufficient condition for asymptotic stability of the zero solution of delay-difference control system of Cellular neural networks in terms of certain matrix inequalities by using a discrete version of the Lyapunov second method. The result is applied to obtain new asymptotic stability condition for some class of delay-difference control system such as delay-difference control system of Cellular neural networks with multiple delays in terms of certain matrix inequalities. Our results can be well suited for computational purposes.

Keywords: Asymptotic stability; Cellular neural networks; Lyapunov function; Delay-difference control system; Matrix inequalities.

1 Introduction

In recent decades, Cellular neural networks have been extensively studied in many aspects and successfully applied to many fields such as pattern identifying, voice recognizing, system controlling, signal processing systems, static image treatment, and solving nonlinear algebraic system, etc. Such applications are based on the existence of equilibrium points, and qualitative properties of systems. In electronic implementation, time delays occur due to some reasons such as circuit integration, switching delays of the amplifiers and communication delays, etc. Therefore, the study of the asymptotic stability of Cellular neural networks with delays is of particular importance to manufacturing high quality microelectronic Cellular neural networks.

While stability analysis of continuous-time neural networks can employ the stability theory of differential system [13], it is much harder to study the stability of discrete-time neural networks [7] with time delays [2] or impulses [9]. The techniques currently available in the literature for discrete-time systems are mostly based on the construction Lyapunov second method [10]. For Lyapunov second method, it is well known that no general rule exists to guide the construction of a proper Lyapunov function for a given system. In fact, the construction of the Lyapunov function becomes a very difficult task.

In this paper, we consider delay-difference control system of Cellular neural networks of the form

$$v(k+1) = -Cv(k) + AS(v(k)) + BS(v(k-h)) + Du(k) + f, \quad (1.1)$$

where $v(k) \in \Omega \subseteq \mathbb{R}^n$ is the neuron state vector, $h \geq 0$, $C = \text{diag}\{c_1, \dots, c_n\}$, $c_i \geq 0$, $i = 1, 2, \dots, n$ is the $n \times n$ constant relaxation matrix, A, B are the $n \times n$ constant weight matrix, D is $n \times m$ constant matrix, $u(k) \in \mathbb{R}^m$ is the control vector, $f = (f_1, \dots, f_n) \in \mathbb{R}^n$ is the constant external input vector and $S(z) = [s_1(z_1), \dots, s_n(z_n)]^T$ with $s_i \in C^1[\mathbb{R}, (-1, 1)]$ where s_i is the neuron activations and monotonically increasing for each $i = 1, 2, \dots, n$.

The asymptotic stability of the zero solution of the delay-differential system of Cellular neural networks has been developed during the past several years. We refer to monographs by CityplaceBurton [3] and Ye [13] and the references cited therein. Much less is known regarding the asymptotic stability of the zero solution of the delay-difference control system of Cellular neural networks. Therefore, the purpose of this paper is to establish sufficient condition for the asymptotic stability of the zero solution of (1.1) in terms of certain matrix inequalities.

2 Preliminaries

The following notations will be used throughout the paper. \mathbb{R}^+ denotes the set of all non-negative real numbers; \mathbb{Z}^+ denotes the set of all non-negative integers; \mathbb{R}^n denotes the n -finite-dimensional Euclidean space with the Euclidean norm $\|\cdot\|$ and the scalar product between x and y is defined by $x^T y$; $\mathbb{R}^{n \times m}$ denotes the set of all $(n \times m)$ -matrices; and A^T denotes the transpose of the matrix A ;

Matrix $Q \in \mathbb{R}^{n \times n}$ is positive semidefinite ($Q \geq 0$) if $x^T Q x \geq 0$, for all $x \in \mathbb{R}^n$. If $x^T Q x > 0$ ($x^T Q x < 0$, resp.) for any $x \neq 0$, then Q is positive (negative, resp.) definite and denoted by $Q > 0$, ($Q < 0$, resp.). It is easy to verify that $Q > 0$, ($Q < 0$, resp.) iff

$$\exists \beta > 0 : x^T Q x \geq \beta \|x\|^2, \forall x \in \mathbb{R}^n,$$

$$(\exists \beta > 0 : x^T Q x \leq -\beta \|x\|^2, \forall x \in \mathbb{R}^n, \text{ resp.})$$

Fact 2.1 For any positive scalar ε and vectors x and y , the following inequality holds:

$$x^T y + y^T x \leq \varepsilon x^T x + \varepsilon^{-1} y^T y.$$

Let us denote $V_\delta = \{x \in \mathbb{R}^n : \|x\| < \delta\}$.

Lemma 2.1 [10] The zero solution of difference system is asymptotic stability if there exists a positive definite function $V(x) : \mathbb{R}^n \rightarrow \mathbb{R}^+$ such that

$$\exists \beta > 0 : \Delta V(x(k)) = V(x(k+1)) - V(x(k)) \leq -\beta \|x(k)\|^2,$$

along the solution of the system. In case the above condition holds for all $x(k) \in V_\delta$, we say that the zero solution is locally asymptotically stable.

We present the following technical lemmas, which will be used in the proof of our main result.

Lemma 2.2 [6] For any constant symmetric matrix $M \in \mathbb{R}^{n \times n}$, $M = M^T > 0$, scalar $s \in \mathbb{R}^+ / \{0\}$, vector function $W : [0, s] \rightarrow \mathbb{R}^n$, we have

$$s \sum_{i=0}^{s-1} (w^T(i) M w(i)) \geq \left(\sum_{i=0}^{s-1} w(i) \right)^T M \left(\sum_{i=0}^{s-1} w(i) \right).$$

3 Main results

In this section, we consider the sufficient condition for asymptotic stability of the zero solution v^* of (1.1) in terms of certain matrix inequalities. Without loss of generality, we can assume that $v^* = 0, S(0) = 0$ and $f=0$ (for otherwise, we let $x = v - v^*$ and define $S(x) = S(x + v^*) - S(v^*)$).

The new form of (1.1) is now given by

$$x(k+1) = -Cx(k) + AS(x(k)) + BS(x(k-h)) + Du(k). \quad (3.1)$$

This is a basic requirement for controller design. Now, we are interested designing a feedback controller for the system (3.1) as

$$u(k) = Kx(k),$$

where K is $n \times m$ constant control gain matrix.

The new form of (3.1) is now given by

$$x(k+1) = -Cx(k) + AS(x(k)) + BS(x(k-h)) + DKx(k). \quad (3.2)$$

Throughout this paper we assume the neuron activations $s_i(x_i)$, $i = 1, 2, \dots, n$ is bounded and monotonically nondecreasing on \mathbb{R} , and $s_i(x_i)$ is Lipschitz continuous, that is, there exist constant $l_i > 0, i = 1, 2, \dots, n$ such that

$$|s_i(r_1) - s_i(r_2)| \leq l_i |r_1 - r_2|, \forall r_1, r_2 \in \mathfrak{R}. \quad (3.3)$$

By condition (3.3), $s_i(x_i)$ satisfy

$$|s_i(x_i)| \leq l_i |x_i|, i = 1, 2, \dots, n. \quad (3.4)$$

Theorem 3.1 *The zero solution of the delay-difference control system (3.2) is asymptotically stable if there exist symmetric positive definite matrices P, G, W and*

$$L = \text{diag}[l_1, \dots, l_n] > 0$$

satisfying the following matrix inequalities:

$$\psi = \begin{pmatrix} (1,1) & 0 & 0 \\ 0 & (2,2) & 0 \\ 0 & 0 & (3,3) \end{pmatrix} < 0 \quad (3.5)$$

where

$$\begin{aligned} (1,1) &= CPC - CPDK - K^T D^T PC + K^T D^T PDK - P + hG + W + \varepsilon CPAA^T PC \\ &+ \varepsilon_1 CPBB^T PC + \varepsilon_2 K^T D^T PAA^T PDK + LA^T PAL + \varepsilon_4 LA^T PBB^T PAL \\ &+ \varepsilon^{-1} LL + \varepsilon_2^{-1} LL, \\ (2,2) &= LB^T PBL + \varepsilon_1^{-1} LL + \varepsilon_3^{-1} LL + \varepsilon_4^{-1} LL - W \end{aligned}$$

and

$$(3,3) = -hG.$$

Proof Consider the Lyapunov function $V(y(k)) = V_1(y(k)) + V_2(y(k)) + V_3(y(k))$, where

$$V_1(y(k)) = x^T(k)Px(k),$$

$$V_2(y(k)) = \sum_{i=k-h+1}^k (h-k+i)x^T(i)Gx(i),$$

$$V_3(y(k)) = \sum_{i=k-h+1}^k x^T(i)Wx(i),$$

P, G , and W being symmetric positive definite solutions of (3.5) and $y(k) = [x(k), x(k-h)]$. Then difference of $V(y(k))$ along trajectory of solution of (3.2) is given by

$$\Delta V(y(k)) = \Delta V_1(y(k)) + \Delta V_2(y(k)) + \Delta V_3(y(k))$$

where

$$\begin{aligned} \Delta V_1(y(k)) &= V_1(x(k+1)) - V_1(x(k)) \\ &= [-Cx(k) + AS(x(k)) + BS(x(k-h)) + DKx(k)]^T P \\ &\times [-Cx(k) + AS(x(k)) + BS(x(k-h)) + DKx(k)] - x^T(k)Px(k) \\ &= x^T(k)[CPC - CPDK - K^T D^T PC + K^T D^T PDK - P]x(k) \\ &- x^T(k)CPAS(x(k)) - S^T(x(k))A^T PCx(k) \\ &- x^T(k)CPBS(x(k-h)) - S^T(x(k-h))B^T PCx(k) \\ &- x^T(k)K^T D^T PAS(x(k)) - S^T(x(k))A^T PDKx(k) \\ &- x^T(k)K^T D^T PBS(x(k-h)) - S^T(x(k-h))B^T PDKx(k) \\ &+ S^T(x(k))A^T PBS(x(k-h)) + S^T(x(k-h))B^T PAS(x(k)) \end{aligned}$$

$$+S^T(x(k))A^T PAS(x(k)) + S^T(x(k-h))B^T PBS(x(k-h)),$$

$$\begin{aligned} \Delta V_2(y(k)) &= \Delta \left(\sum_{i=k-h+1}^k (h-k+i)x^T(i)Gx(i) \right) \\ &= hx^T(k)Gx(k) - \sum_{i=k-h+1}^k x^T(i)Gx(i) \end{aligned}$$

and

$$\begin{aligned} \Delta V_3(y(k)) &= \Delta \left(\sum_{i=k-h+1}^k x^T(i)Wx(i) \right) \\ &= x^T(k)Wx(k) - x^T(k-h)Wx(k-h), \end{aligned} \tag{3.6}$$

where (3.4) and **Fact 2.1** are utilized in (3.6), respectively.

Note that

$$\begin{aligned} &-x^T(k)CPAS(x(k)) - S^T(x(k))A^T PCx(k) \\ &\quad \leq \varepsilon x^T(k)CPAA^T PCx(k) + \varepsilon^{-1}S^T(x(k))S(x(k)), \\ &-x^T(k)CPBS(x(k-h)) - S^T(x(k-h))B^T PCx(k) \\ &\quad \leq \varepsilon_1 x^T(k)CPBB^T PCx(k) + \varepsilon_1^{-1}S^T(x(k-h))S(x(k-h)), \\ &-x^T(k)K^T D^T PAS(x(k)) - S^T(x(k))A^T PDKx(k) \\ &\quad \leq \varepsilon_2 x^T(k)K^T D^T PAA^T PDKx(k) + \varepsilon_2^{-1}S^T(x(k))S(x(k)), \\ &-x^T(k)K^T D^T PBS(x(k-h)) - S^T(x(k-h))B^T PDKx(k) \\ &\quad \leq \varepsilon_3 x^T(k)K^T D^T PBB^T PDKx(k) + \varepsilon_3^{-1}S^T(x(k-h))S(x(k-h)), \\ &S^T(x(k))A^T PBS(x(k-h)) + S^T(x(k-h))B^T PAS(x(k)) \\ &\quad \leq \varepsilon_4 S^T(k)A^T PBB^T PAS(k) + \varepsilon_4^{-1}S^T(x(k-h))S(x(k-h)), \\ &S^T(x(k-h))B^T PBS(x(k-h)) \leq x^T(k-h)LB^T PBLx(k-h), \\ &S^T(x(k))A^T PAS(x(k)) \leq x^T(k)LA^T PALx(k), \\ &\varepsilon_4 S^T(k)A^T PBB^T PAS(k) \leq \varepsilon_4 x^T(k)LA^T PBB^T PALx(k), \\ &\varepsilon_1^{-1}S^T(x(k-h))S(x(k-h)) \leq \varepsilon_1^{-1}x^T(k-h)LLx(k-h), \\ &\varepsilon_3^{-1}S^T(x(k-h))S(x(k-h)) \leq \varepsilon_3^{-1}x^T(k-h)LLx(k-h), \\ &\varepsilon_4^{-1}S^T(x(k-h))S(x(k-h)) \leq \varepsilon_4^{-1}x^T(k-h)LLx(k-h), \\ &\varepsilon^{-1}S^T(x(k))S(x(k)) \leq \varepsilon^{-1}x^T(k)LLx(k) \end{aligned}$$

and

$$\varepsilon_2^{-1}S^T(x(k))S(x(k)) \leq \varepsilon_2^{-1}x^T(k)LLx(k),$$

hence

$$\begin{aligned} \Delta V_1 \leq & x^T(k)[CPC - CPDK - K^T D^T PC + K^T D^T PDK - P]x(k) + \varepsilon x^T(k)CPAA^T PCx(k) \\ & + \varepsilon_1 x^T(k)CPBB^T PCx(k) + \varepsilon_2 x^T(k)K^T D^T PAA^T PDKx(k) + x^T(k)LA^T PALx(k) \\ & + \varepsilon_4 x^T(k)LA^T PBB^T PALx(k) + \varepsilon^{-1} x^T(k)LLx(k) + \varepsilon_2^{-1} x^T(k)LLx(k) \\ & + x^T(k-h)LB^T PBLx(k-h) + \varepsilon_1^{-1} x^T(k-h)LLx(k-h) + \varepsilon_3^{-1} x^T(k-h)LLx(k-h) \\ & + \varepsilon_4^{-1} x^T(k-h)LLx(k-h). \end{aligned}$$

Then we have

$$\begin{aligned} \Delta V \leq & x^T(k)[CPC - CPDK - K^T D^T PC + K^T D^T PDK - P + hG + W + \varepsilon CPAA^T PC \\ & + \varepsilon_1 CPBB^T PC + \varepsilon_2 K^T D^T PAA^T PDK + LA^T PAL + \varepsilon_4 LA^T PBB^T PAL \\ & + \varepsilon^{-1} LL + \varepsilon_2^{-1} LL]x(k) \\ & + x^T(k-h)[LB^T PBL + \varepsilon_1^{-1} LL + \varepsilon_3^{-1} LL + \varepsilon_4^{-1} LL - W]x(k-h) \\ & - \sum_{i=k-h}^{k-1} x^T(i)Gx(i). \end{aligned}$$

Using **Lemma 2.2**, we obtain

$$\sum_{i=k-h}^{k-1} x^T(i)Gx(i) \geq \left(\frac{1}{h} \sum_{i=k-h}^{k-1} x(i) \right)^T (hG) \left(\frac{1}{h} \sum_{i=k-h}^{k-1} x(i) \right).$$

From the above inequality it follows that:

$$\begin{aligned} \Delta V \leq & x^T(k)[CPC - CPDK - K^T D^T PC + K^T D^T PDK - P + hG + W + \varepsilon CPAA^T PC \\ & + \varepsilon_1 CPBB^T PC + \varepsilon_2 K^T D^T PAA^T PDK + LA^T PAL + \varepsilon_4 LA^T PBB^T PAL \\ & + \varepsilon^{-1} LL + \varepsilon_2^{-1} LL]x(k) \\ & + x^T(k-h)[LB^T PBL + \varepsilon_1^{-1} LL + \varepsilon_3^{-1} LL + \varepsilon_4^{-1} LL - W]x(k-h) \\ & - \left(\frac{1}{h} \sum_{i=k-h}^{k-1} x(i) \right)^T (hG) \left(\frac{1}{h} \sum_{i=k-h}^{k-1} x(i) \right) \\ = & \left(x^T(k), x^T(k-h), \left(\frac{1}{h} \sum_{i=k-h}^{k-1} x(i) \right)^T \right) \begin{pmatrix} (1,1) & 0 & 0 \\ 0 & (2,2) & 0 \\ 0 & 0 & (3,3) \end{pmatrix} \begin{pmatrix} x(k) \\ x(k-h) \\ \left(\frac{1}{h} \sum_{i=k-h}^{k-1} x(i) \right) \end{pmatrix} \\ = & y^T(k)\psi y(k), \end{aligned}$$

where

$$\begin{aligned} (1,1) = & CPC - CPDK - K^T D^T PC + K^T D^T PDK - P + hG + W + \varepsilon CPAA^T PC \\ & + \varepsilon_1 CPBB^T PC + \varepsilon_2 K^T D^T PAA^T PDK + LA^T PAL + \varepsilon_4 LA^T PBB^T PAL \end{aligned}$$

$$+\varepsilon^{-1}LL + \varepsilon_2^{-1}LL,$$

$$(2, 2) = LB^T PBL + \varepsilon_1^{-1}LL + \varepsilon_3^{-1}LL + \varepsilon_4^{-1}LL - W,$$

$$(3, 3) = -hG,$$

$$\text{and } y(k) = \begin{pmatrix} x(k) \\ x(k-h) \\ \left(\frac{1}{h} \sum_{i=k-h}^{k-1} x(i)\right) \end{pmatrix}.$$

By the condition (3.5), $\Delta V(y(k))$ is negative definite, namely there is a number $\beta > 0$ such that $\Delta V(y(k)) \leq -\beta \|y(k)\|^2$, and hence, the asymptotic stability of the system immediately follows from **Lemma 2.1**. This completes the proof. \square

Remark 3.1 Theorem 3.1 gives a sufficient condition for the asymptotic stability of delay-difference system (3.1) via matrix inequalities. These conditions are described in terms of certain diagonal matrix inequalities, which can be realized by using the linear matrix inequality algorithm proposed in [4]. But Hu and Wang [9] these conditions are described in terms of certain symmetric matrix inequalities, which can be realized by using the Schur complement lemma and linear matrix inequality algorithm proposed in [4].

Example 3.1 Let us consider a delay-difference control system of Cellular neural networks (3.2), given by the system

$$x(k+1) = -Ax(k) + BS(x(k-h)) + CKx(k),$$

where the matrices are

$$C = \begin{pmatrix} 3.5 & 0 \\ 0 & 1.8 \end{pmatrix}, A = \begin{pmatrix} 0.7339 & -0.0196 \\ 0.0326 & 0.5412 \end{pmatrix}, B = \begin{pmatrix} 0.5588 & -0.0824 \\ 0.0706 & 0.7412 \end{pmatrix}, D = \begin{pmatrix} 1 \\ 0 \end{pmatrix},$$

$$s_i(x_i) = \frac{2}{\pi} \tan^{-1}(x_i), i = 1, 2, \varepsilon = 0.5, \varepsilon_1 = 0.5, \varepsilon_2 = 0.5, \varepsilon_3 = 0.5, \varepsilon_4 = 0.5, \text{ and } h = 1.$$

Using the LMI Toolbox in MATLAB, we found that the LMIs in Theorem 3.1 are feasible and

$$P = \begin{pmatrix} 0.5863 & 0.0893 \\ 0.0893 & 0.8532 \end{pmatrix}, G = \begin{pmatrix} 5.6983 & 1.2369 \\ 1.2369 & 6.2354 \end{pmatrix}, W = \begin{pmatrix} 0.5968 & 0.2596 \\ 0.2596 & 0.9386 \end{pmatrix},$$

$$L = \begin{pmatrix} 3.6393 & 0 \\ 0 & 0.2586 \end{pmatrix}, K = (7.8963 \quad 8.3257) \text{ are set of solutions to the LMIs (3.5).}$$

By a straightforward, we have

$$\psi = \begin{pmatrix} -0.0245 & 0 \\ 0 & -0.1896 \end{pmatrix}.$$

The eigenvalues are -0.0245 and -0.1896, respectively. This implies the matrix $\psi < 0$. It follows from Lemma 2.1 that the zero solution of delay-difference control system of Cellular neural networks is asymptotically stable.

4 Applications

In this section, we apply the main result of this paper, which provides a sufficient condition for the asymptotic stability of delay-difference control system of Cellular neural networks with multiple delays in terms of certain matrix inequalities.

We consider delay-difference control system of Cellular neural networks with multiple delays of the form

$$v(k+1) = -Cv(k) + AS(v(k)) + \sum_{i=1}^m B_i S(v(k-h_i)) + Du(k) + f, \quad (4.1)$$

where $v(k) \in \Omega \subseteq \mathbb{R}^n$ is the neuron state vector, $0 \leq h_1 \leq \dots \leq h_m$, $C = \text{diag}\{c_1, \dots, c_n\}$, $c_i \geq 0$, $i = 1, 2, \dots, n$ is the $n \times n$ constant relaxation matrix, B, B_i , $i = 1, 2, \dots, m$ are the $n \times n$ constant weight matrices, D is $n \times m$ constant matrix, $u(k) \in \mathbb{R}^m$ is the control vector, $f = (f_1, \dots, f_n) \in \mathbb{R}^n$ is the constant external input vector and $S(z) = [s_1(z_1), \dots, s_n(z_n)]^T$ with $s_i \in C^1[\mathbb{R}, (-1, 1)]$ where s_i is the neuron activations and monotonically increasing for each $i = 1, 2, \dots, n$.

We consider the sufficient condition for asymptotic stability of the zero solution v^* of (4.1) in terms of certain matrix inequalities. Without loss of generality, we can assume that $v^* = 0, S(0) = 0$ and $f=0$ (for otherwise, we let $x = v - v^*$ and define $S(x) = S(x + v^*) - S(v^*)$).

The new form of (4.1) is now given by

$$x(k+1) = -Cx(k) + AS(x(k)) + \sum_{i=1}^m B_i S(x(k-h_i)) + Du(k). \quad (4.2)$$

This is a basic requirement for controller design. Now, we are interested designing a feedback controller for the system (4.2) as

$$u(k) = Kx(k),$$

where K is $n \times m$ constant control gain matrix.

The new form of (4.2) is now given by

$$x(k+1) = -Cx(k) + AS(x(k)) + \sum_{i=1}^m B_i S(x(k-h_i)) + DKx(k) \quad (4.3)$$

Theorem 4.1 *The zero solution of the delay-difference system of Cellular neural networks with multiple delays (4.3) is asymptotically stable if there exist symmetric positive definite matrices P, G_i, W_i , and $L = \text{diag}[l_1, \dots, l_m] > 0$, $i = 1, 2, \dots, m$ satisfying the following matrix inequalities:*

$$\psi < O \quad (4.4)$$

where

$$\psi = \begin{bmatrix} (0,0) & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & (1,1) & (1,2) & \cdots & (1,m) & 0 & 0 & 0 & \cdots & 0 \\ 0 & (2,1) & (2,2) & \cdots & (2,m) & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & (m,1) & (m,2) & \cdots & (m,m) & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & (m+1,m+1) & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & (m+2,m+2) & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & (2m,2m) \end{bmatrix}$$

and,

$$(0,0) = CPC - CPDK - K^T D^T PC + K^T D^T PDK - P + \sum_{i=1}^m (h_i G_i + W_i)$$

$$+ \varepsilon CPAA^T PC + \varepsilon_1 \sum_{i=1}^m \sum_{j=1}^m CPB_i B_j^T PC + \varepsilon_2 K^T D^T PAA^T PDK + LA^T PAL$$

$$+ \varepsilon_4 \sum_{i=1}^m \sum_{j=1}^m LA^T PB_i B_j^T PAL + \varepsilon^{-1} LL + \varepsilon_2^{-1} LL,$$

$$(1,1) = LB_1^T PB_1 L + \varepsilon_1^{-1} LL + \varepsilon_3^{-1} LL + \varepsilon_4^{-1} LL - W_1,$$

$$(1,2) = LB_1^T PB_2 L + \varepsilon_1^{-1} LL + \varepsilon_3^{-1} LL + \varepsilon_4^{-1} LL,$$

$$(1,m) = LB_1^T PB_m L + \varepsilon_1^{-1} LL + \varepsilon_3^{-1} LL + \varepsilon_4^{-1} LL,$$

$$(2,1) = LB_2^T PB_1 L + \varepsilon_1^{-1} LL + \varepsilon_3^{-1} LL + \varepsilon_4^{-1} LL,$$

$$(2,2) = LB_2^T PB_2 L + \varepsilon_1^{-1} LL + \varepsilon_3^{-1} LL + \varepsilon_4^{-1} LL - W_2,$$

$$(2,m) = LB_2^T PB_m L + \varepsilon_1^{-1} LL + \varepsilon_3^{-1} LL + \varepsilon_4^{-1} LL,$$

$$(m,1) = LB_m^T PB_1 L + \varepsilon_1^{-1} LL + \varepsilon_3^{-1} LL + \varepsilon_4^{-1} LL,$$

$$(m,2) = LB_m^T PB_2 L + \varepsilon_1^{-1} LL + \varepsilon_3^{-1} LL + \varepsilon_4^{-1} LL,$$

$$(m,m) = LB_m^T PB_m L + \varepsilon_1^{-1} LL + \varepsilon_3^{-1} LL + \varepsilon_4^{-1} LL - W_m,$$

$$(m+1, m+1) = -h_1 G_1,$$

$$(m+2, m+2) = -h_2 G_2, \text{ and}$$

$$(2m, 2m) = -h_m G_m.$$

Proof Consider the Lyapunov function $V(y(k)) = V_1(y(k)) + V_2(y(k)) + V_3(y(k))$, where

$$V_1(y(k)) = x^T(k) P x(k),$$

$$V_2(y(k)) = \sum_{i=1}^m \sum_{j=k-h_i+1}^k (h-k+i) x^T(j) G_i x(j),$$

$$V_3(y(k)) = \sum_{i=1}^m \sum_{j=k-h_i+1}^k x^T(j)W_i x(j),$$

$P, G_i,$ and $W_i, i = 1, 2, \dots, m$ being symmetric positive definite solutions of (4.4) and $y(k) = [x(k), x(k-h_1), \dots, x(k-h_m)]$.

Then difference of $V(y(k))$ along trajectory of solution of (4.3) is given by

$\Delta V(y(k)) = \Delta V_1(y(k)) + \Delta V_2(y(k)) + \Delta V_3(y(k))$, where

$$\Delta V_1 = V_1(x(k+1)) - V_1(x(k))$$

$$= [-Cx(k) + AS(x(k)) + \sum_{i=1}^m B_i S(x(k-h_i)) + DKx(k)]^T P$$

$$\times [-Cx(k) + AS(x(k)) + \sum_{i=1}^m B_i S(x(k-h_i)) + DKx(k)]$$

$$-x^T(k)Px(k) = x^T(k)[CPC - CPDK - K^T D^T PC + K^T D^T PDK - P]x(k)$$

$$-x^T(k)CPAS(x(k)) - S^T(x(k))A^T PCx(k) - \sum_{i=1}^m x^T(k)CPB_i S(x(k-h_i))$$

$$- \sum_{i=1}^m S^T(x(k-h_i))B_i^T PCx(k) - x^T(k)K^T D^T PAS(x(k)) - S^T(x(k))A^T PDKx(k)$$

$$- \sum_{i=1}^m x^T(k)K^T D^T PB_i S(x(k-h_i)) - \sum_{i=1}^m S^T(x(k-h_i))B_i^T PDKx(k)$$

$$+ \sum_{i=1}^m S^T(x(k))A^T PB_i S(x(k-h_i)) + \sum_{i=1}^m S^T(x(k-h_i))B_i^T PAS(x(k))$$

$$+ S^T(x(k))A^T PAS(x(k)) + \sum_{i=1}^m \sum_{j=1}^m S^T(x(k-h_i))B_i^T PB_j S(x(k-h_j)),$$

$$\Delta V_3 = \Delta \left(\sum_{i=1}^m \sum_{j=k-h_i}^{k-1} x^T(j)W_i x(j) \right) = \sum_{i=1}^m x^T(k)W_i x(k) - \sum_{i=1}^m x^T(k-h_i)W_i x(k-h_i)$$

The rest of the proof is similar to that of **Theorem 3.1** need hold. \square

5 Conclusions

In this paper, based on a discrete analog of the Lyapunov second method, we have established a sufficient condition for the asymptotic stability of delay-difference control system of Cellular neural networks in terms of certain matrix inequalities. The result has been applied to obtain new stability condition for some class of delay-difference control system such as delay-difference control system of Cellular neural networks with multiple delays in terms of certain matrix inequalities.

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